Three and Four Carriers for Reliable Ambiguity Resolution

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BIOGRAPHY

Dr. Knut Sauer received a Ph.D. in Satellite Navigation from the Imperial College of Science, Medicine and Technology, London. 2003 he joined Trimble Terrasat as software and development engineer where he is working on high precision kinematic positioning using the future Galileo system.

Dr. Ulrich Vollath received a Ph.D. in Computer Science from the Munich University of Technology (TUM) in 1993. At Trimble Terrasat - where he is working on GPS algorithms since more than eleven years - he is responsible for the Algorithm Development team. His professional interest is focused on high-precision real-time kinematic positioning and reference station network processing.

Francisco Amarillo is the ESA/ESTEC technical officer responsible for the investigation on four carrier ambiguity resolution.

ABSTRACT

The use of three carriers to perform high precision carrier phase based positioning now has been accepted widely for the modernized GPS as well as for the planned Galileo satellite navigation system. In principle, instantaneous (one-epoch) ambiguity resolution becomes feasible for a broad range of applications. A boost of system availability and reliability is recognized as well.

For Galileo, in addition to the open access service (OS) a commercial service (PRS) is planned. Besides other improvements a fourth carrier frequency E6 will be available. It has to be proved that the corresponding fees are justified by the actual gain in service quality.

Based on a contract with ESA/ESTEC, an extensive hardware-simulation of the planned three- and four-carrier options for Galileo/GPS was carried out. A modified Hardware satellite-signal-simulator provided GNSS data containing three or four carrier-frequencies which were tracked by an AGGA-2 receiver. The range of simulation parameters covers various levels of ionosphere, troposphere and multipath. Up to 10 GPS or Galileo satellites were tracked simultaneously on baselines up to 82km. The simulation scenarios cover surface, airborne and static applications.

The new Factorized Multi-Carrier Ambiguity Resolution (FAMCAR) approach for efficient combination of multi-carrier data was used to analyze OTF ambiguity resolution performance.

This paper presents final experimental results including detailed analyses of reliability, availability and accuracy of ambiguity resolution and carrier based positioning for the generated data sets using new processing algorithms. The influence of major error sources and signal design on the system performance is evaluated in depth. Furthermore the statistical significance of such experiments, especially with the scope on high reliability-application is discussed briefly.

A final comparison between the two, three and four carrier solutions concludes the paper. It reveals significant improvements especially for baselines over 35 km comparing the two- and the three-frequency solution. The benefit of the fourth carrier is less pronounced, but the effort of providing it may still be justified for applications with very high requirements on reliability and availability.

The impacts of multipath and ionosphere (for longer distances to the reference station) are mitigated by the use of at least three carriers. Still, they remain to be the limiting factors for carrier-based positioning performance.

INTRODUCTION

To emulate performance analyses near to reality, a hardware simulation of the new signals was performed under an ESA/ESTEC contract. This project is the follow-up of experiments investigating three-carrier ambiguity resolution ([Vollath et. al. 1998], [Vollath et. al. 2001]). The main purpose of the experiment was to investigate if the benefits of a fourth carrier justify a commercial pay-service. Also, differences in the expected performance of modernized GPS and Galileo were of interest.

The presented paper gives a brief overview on the actual experiment performed at the European Space and Technology Centre (ESTEC).

The actual processing algorithm combing the various observation types in an optimal manner is discussed briefly in the second Section.

The choice of the used power level for the simulations to meet the code-noise requirements did not allow realistic carrier-phase multipath. For this particular reason a first-order Gauss-Markov process was implemented to simulate realistic carrier-phase multipath. This approach is discussed in the third section.

The fourth section summarizes the processing steps being performed to evaluate the performance.

The fifth section contains a short excurse on statistical significance especially for applications were levels of reliability in the order of $<10^{-4}$ are required.

The last section contains the final results of the study. Performance parameters such as fixing reliability, mean Time-to-Fix and positioning accuracy versus baseline length, carrier-phase multipath and ionospheric delay are presented.

TEST SET-UP

For the simulations, the following equipment was used:

- A Spirent STS Series Multi Channel Simulator, modified to transmit GPS codes on GLONASS channels
- A breadboard receiver capable of tracking 6 satellites simultaneously on three carriers.
- An atomic frequency standard
- A Pulse-per-Second synchronization board to line-up receiver and signal generator time
- A workstation for receiver control and data logging

The block diagram of the hardware test equipment is shown in Figure 1.

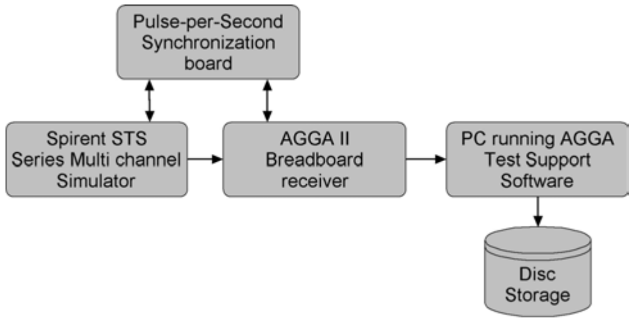

Figure 1: Test hardware set-up

Table 1 shows the simulated and the actual/planned frequencies as used in the experiment. The GLONASS channels have been assigned in order to emulate the frequency spacing as close as possible.

Table 1: Summary of simulated and allocated frequencies for Galileo and for GPS Frequency

Galileo				
Simulated Frequencies	L1 (1575,42 MHz)	L2 (1227,60 MHz)	G(7) (1605,9375 MHz)	G(24) (1615,50 MHz)
Actual/ Planned Frequencies	L1 (1575.42 MHz)	E5a (1176.45 MHz)	E5b (1207.14 MHz)	E6 (1278.75 MHz)
GPS				
Simulated Frequencies	L1 (1575,42 MHz)	L2 (1227,60 MHz)	G(24) (1615,50 MHz)	n.a.
Actual/ Planned Frequencies	L1 (1575,42 MHz)	L2 (1227,60 MHz)	L5 (1176.45 MHz)	n.a.

Especially, GPS L5 and Galileo E5b were simulated with GLONASS channel 7, Galileo E6 using GLONASS channel 24. All codes were tracked as P-Codes to guarantee the low noise and multipath values required.

To be able to generate four carriers and more than 6 satellites, every data set was combined from 4 hardware simulations. The combination of different runs requires tight synchronization of the receive times. For that reason, a Pulse-per-Second (PPS) synchronization board was used to align the signal simulator and the receiver and assuring tracking at pre-programmed reception times.

The remaining error in the receiver clocks was calibrated in the combination process. The combination procedure is shown in Figure 2.

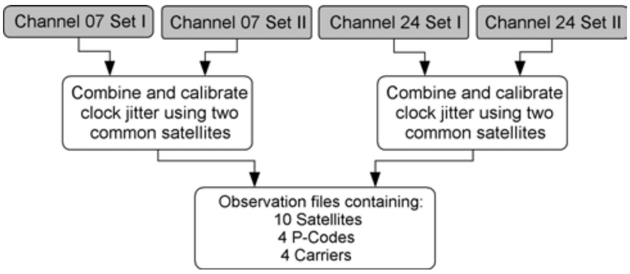

Figure 2: Data file combination

After the experimental part was successfully completed at ESTEC an extensive data quality and performance analysis was performed.

FACTORIZED MULTI-CARRIER AMBIGUITY RESOLUTION (FAMCAR)

Brief insight in the Factorized Multi-Carrier Ambiguity Resolution (FAMCAR) algorithm shall be given here. A detailed description and derivation may be found in [Vollath 2004].

The concept introduces a number of new independent linear combination of carrier-phase observations as well as of carrier-phase and pseudo-range observations. The combinations include the minimum-error geometric carrier-phase combination, the minimum-error ionosphere combination, the new Quintessence combinations and the code-carrier combinations. From these individual estimates, the full floating solution for all carriers is derived (see Figure 3).

Existing standard techniques for multi-carrier ambiguity determination usually apply one big Kalman filter to estimate all unknowns (e.g. position, ambiguities, ionosphere and multipath). The factorization enables the stepwise modeling of each error component and leads therefore to a bank of significantly smaller filters. This approach results in distinct higher computational efficiency for the Kalman filter sets (i.e. float solution) and a better knowledge of each error component for the individual measurements. In addition to the efficient processing of three and four carrier data the new approach is already applicable to a dual-frequency system. Furthermore the decreased computational load enables the use of smaller processor components and therefore provides a significant cost reduction.

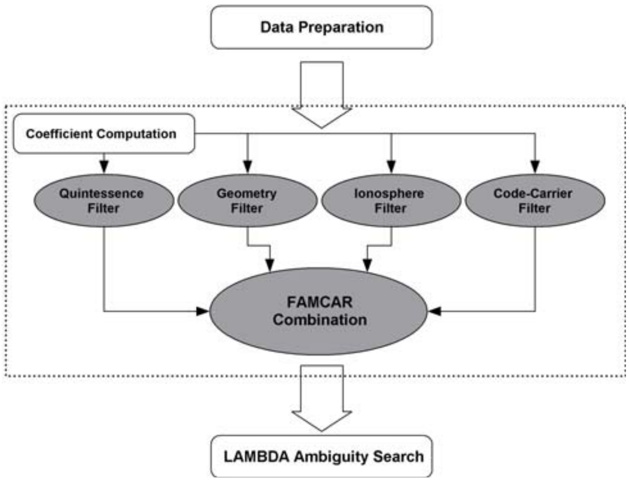

Figure 3: Overview on dataflow for Factorized Multi-Carrier Ambiguity Resolution

CARRIER-PHASE MULTIPATH GENERATION

Due to the required levels of code noise in the hardware simulation a power level had to be chosen so that no significant carrier-phase multipath was simulated. The main reason is that the AGGA breadboard receiver used does not implement any high performance multipath mitigation techniques at hardware/firmware level.

Nevertheless realistic satellite observations contain a certain amount of code as well as carrier-phase multipath. The order of magnitude is usually somewhat correlated. The absence of carrier-phase multipath would skew that relation.

Furthermore carrier-phase multipath has a great influence on the performance of ambiguity fixing in real-time. To add realistic carrier-phase multipath during runtime the processing engine was adapted accordingly. Therefore a time-correlated Gauss-Markov 1st order process has been implemented. The parameters to be specified are the time constant *t* and the elevation-weighted a-priori variance of the multipath $\sigma^2_{MP,n}$. Based on these two input parameters the carrier-phase multipath is computed with the correlation time t_c.

$$t_c = e^{-1/t}$$

The variance of the time-correlated process σ^2_{tc} has been computed as follows:

$$\sigma^2_{tc} = \left(1 - t_c^2\right) \cdot \sigma^2_{MP,n}$$

Finally the values GM_k at epoch *k* of time series for the Gauss-Markov process are computed and added separately for each carrier as follows:

For k = 0

$$GM_k = RAND \cdot \sqrt{\sigma^2_{MP,n}}$$

For k > 0

$$GM_k = t_c \cdot \sqrt{\frac{\sigma^2_{MP,n}}{\sigma^2_{MP,n-1}}} \cdot GM_{k-1} + RAND \cdot \sqrt{\sigma^2_{tc}}$$

The time series in Figure 4 shows an example for the generated carrier-phase multipath on L1 for PRN 13.

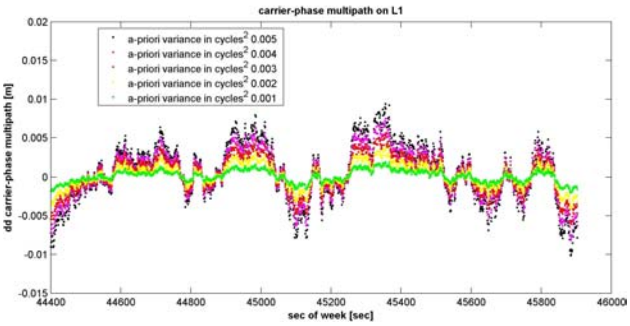

Figure 4: Time series of carrier-phase multipath based on various a-priori variance levels

The resulting LC (ionosphere-free) residuals are shown in Figure 5.

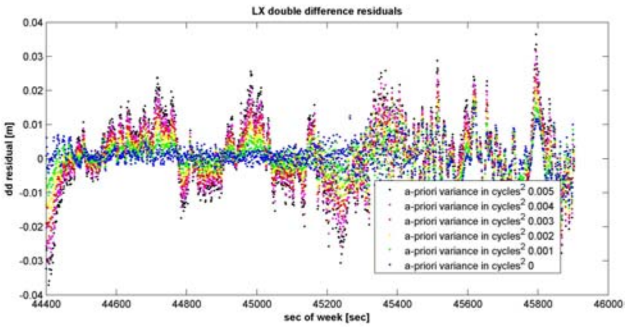

Figure 5: Time series of double-difference LX residuals based on various levels of a-priori multipath variance

Throughout the study five levels from 0.001… 0.005 $cycl^2$ and a correlation time of 50 second have been used for both systems to simulate the carrier-phase.

DATA PROCESSING

The final data processing included an extensive quality assessment for the simulated data sets as well as an extensive processing of the combined dataset considering parameters as carrier-phase multipath and ionospheric error component.

The following Table 2 summarizes the data quality parameters as determined by the quality assessment

Table 2: Final results of data quality assessment

parameter	GPS		Galileo	
	without multipath	with multipath	without multipath	with multipath
$RMS_{carrier}$	~0.4mm	~0.5… 1.0 mm	~0.4mm	~0.5… 1.0 mm
RMS_{code}	~7.0 cm	~22.0 cm	~5.0 cm	~20.0 cm
t_c (carrier multipath)	na	50 sec	na	50 sec
$\sigma_{MP\text{-}carrr}$ [$cycl^2$]		0.001… 0.005		0.001… 0.005

The processing was performed on all scenarios on five different levels of carrier-phase multipath and ten different levels of ionospheric error component. The total number of analyzed static and kinematic baselines is 45. For each baseline approximately 1200 epochs (1Hz) have been processed. This may seem a lot, but for applications where high levels of reliability are required ($<10^{-4}$) it is by far not sufficient.

The following note may give a little insight in the statistical significance of such experiments.

NOTE ON STATISTICAL SIGNIFICANCE

For the presented type of analyses it is very crucial to assess the statistical significance of the results obtained. As the data length for every simulated session is fairly limited (1500 epochs) due to the constraint that generation has to be performed in real time, the question is which levels of reliability can be significantly distinguished for which data of that size.

For an analytical solution, one assumption has to be done. Every fixing attempt is assumed to be independent to each other. While the errors sources are time correlated, i.e. multipath and ionosphere, and the different fixing attempts even share data. Still this assumption is not too unrealistic as the multipath and ionosphere errors are actually modeled in the ambiguity resolution method used.

The study assesses this aspect in some detail. Therefore it is assumed that for these types of experiments the *binomial distribution* describes the statistical properties [Spiegel et. al. 1975]. The following table shows the minimum differences in the number of wrong fixes required to falsify the hypothesis of identical failure probabilities with a confidence level of 95% for the typically 1500 samples used for a baseline in this analysis.

Table 3: Significance of difference failure rates

Number of wrong fixes	Failure Probability [%]	Calculated Reliability [%]	Min. significantly different #	Failure Probability [%]	Effective Reliability [%]
0	0.00	100.00	4	0.26	99.74
1	0.07	99.93	7	0.47	99.53
2	0.13	99.87	9	0.60	99.40
4	0.27	99.73	12	0.80	99.20
6	0.40	99.60	15	1.00	99.00
8	0.53	99.47	18	1.20	98.80

10	0.67	99.33	21	1.40	98.60
14	0.93	99.07	27	1.80	98.20
18	1.20	99.80	32	2.13	97.87
22	1.47	98.53	37	2.47	97.53
26	1.73	98.27	42	2.80	97.20
30	2.00	98.00	47	3.13	96.87

This implies for example, that a baseline scenario with no wrong fixes cannot be statistically significant (95% confidence level) be distinguished from one with three wrong fixes (0.2% failure rate, 99.8 % success rate). In other words, no wrong fixes prove only a reliability of better than 99.74 %.

PERFORMANCE ANALYSIS

The following step in the project was to evaluate the performance in terms of ambiguity fixing and positioning. Therefore the following performance parameters have been specified for the fixing performance:

- Reliability [%]: This denotes the ratio between the "good fixes" (i.e. correct fixes) and the "bad fixes" (i.e. detected wrong fixes).
- Mean Time-to-Fix [sec]: This denotes the averaged time the software needs to determine the correct ambiguity within the time window.
- Min Time-to-Fix [sec]: This denotes the minimum time the software needs to determine the correct ambiguity within the time window.
- Max Time-to-Fix [sec]: This denotes the maximal time the software needs to determine the correct ambiguity within the time window.
- Time-to-Fix, 90% [sec]: This denotes the time the software needs in 90 percent of all attempts to determine the correct ambiguity within the time window.
- Percent inst. Fixing [%]: This denotes the percentage the software fixes the ambiguities within the first epoch.

The performance in terms of positioning accuracy was determined by comparing the computed positions and trajectories with the their true counterparts. Out of these time series the resulting RMS values for the north, east and the height component have been derived.

To evaluate the performance in respect to carrier-phase multipath, ionospheric error component and baseline length the following examples shall be given.

a) Reliability vs. Carrier-Multipath and Double Difference Ionospheric Residual

These plots visualize the correlation between reliability (i.e. ratio between good and bad fixes) the *a-priori* carrier-phase multipath level and the maximal double-difference ionospheric residual. Shown is an example of a 32 kilometer baseline for Galileo and the corresponding dual-, tree- and the four-frequency solution. The result shows the significant correlation between the input parameters and the computed reliability.

Furthermore the examples in Figure 6 to Figure 8 illustrate the significant performance gain comparing dual (DCAR) and three-frequency (TCAR) processing. It can be seen that the gain in terms of reliability comparing three and four frequency (FCAR) processing is less pronounced.

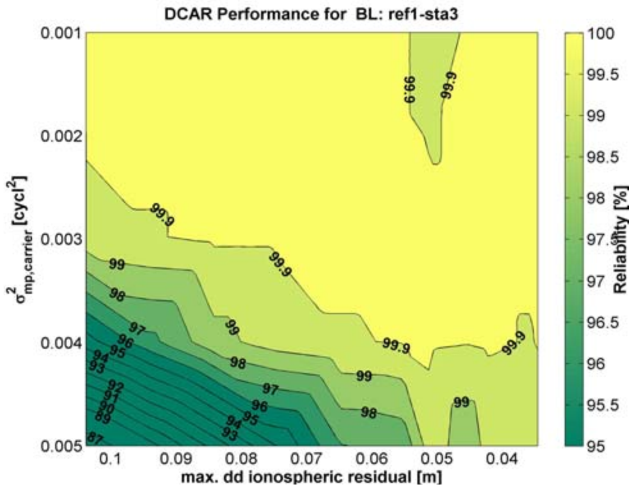

Figure 6: DCAR performance – Reliability vs. a-priori multipath and max dd-iono residual– Galileo - BL REF1-STA3 (32KM)

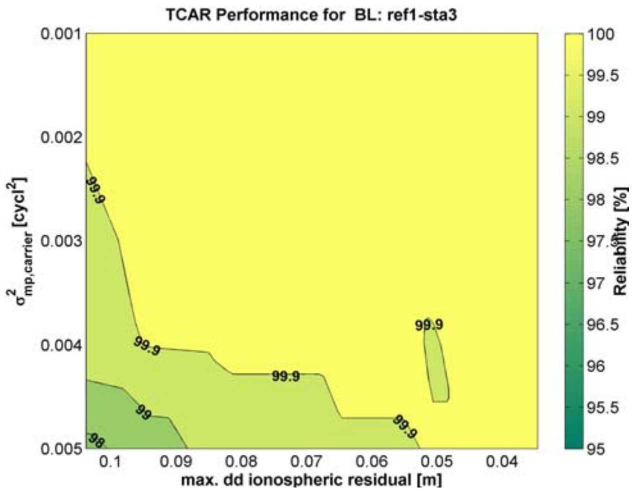

Figure 7: TCAR performance – Reliability vs. a-priori multipath and max dd-iono residual– Galileo - BL REF1-STA3 (32KM)

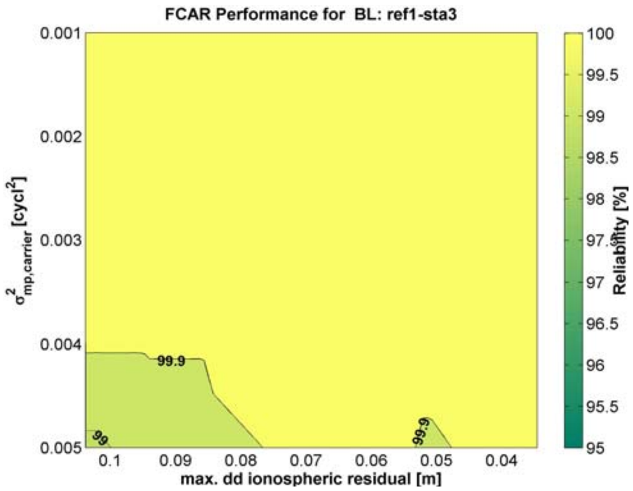

Figure 8: FCAR performance – Reliability vs. a-priori multipath and max dd-iono residual– Galileo - BL REF1-STA3 (32KM)

b) Reliability vs. Baseline length

These plots visualize the correlation between the computed reliability and the corresponding baseline length. A clear dependency can be seen in Figure 9 and Figure 10. Shown are plots of the final computed reliability versus the baseline length for GPS (dual and three carrier solution) and for Galileo (dual, three and four carrier solution). This shows that with an increasing baseline length the share of un-modeled error components increases as well i.e. the reliability drops. In both the GPS and the Galileo scenarios the reliability increases significantly comparing the dual- and the three-frequency solution. Again the difference between- the three and the four-frequency solution is less significant.

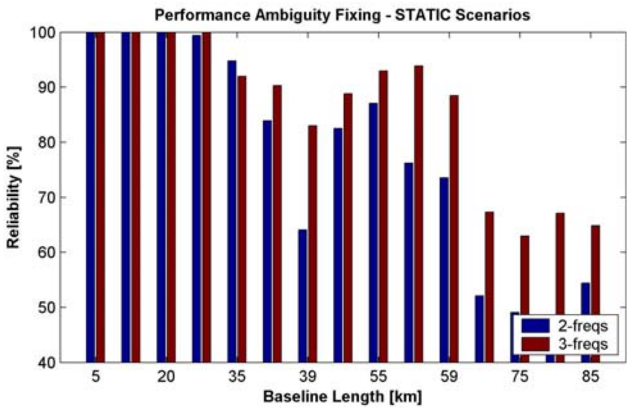

Figure 9: GPS Fixing Performance (reliability) vs. baseline length

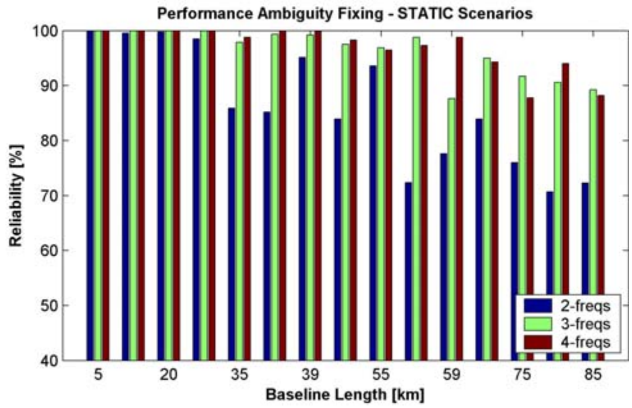

Figure 10: Galileo Fixing Performance (reliability) vs. baseline length

c) Time-to-Fix vs. Baseline length

The third analysis shows the correlation between the baseline length and the mean time-to-fix. This is shown for both GPS and Galileo. A significant gain can be recognized comparing the dual- with the three-carrier solution. A less significant gain can be seen comparing the three- and the four-carrier solution.

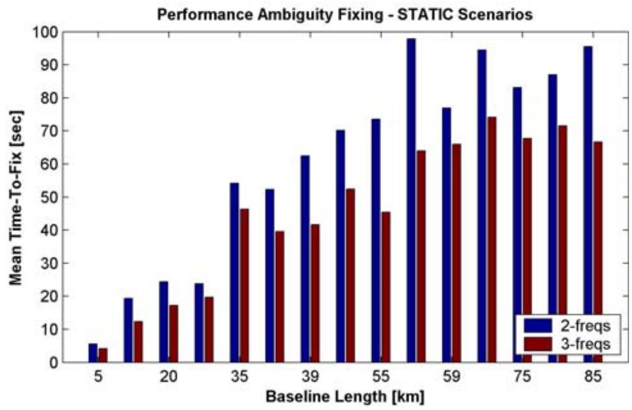

Figure 11: GPS Fixing Performance (mean-Time-to-Fix) vs. baseline length

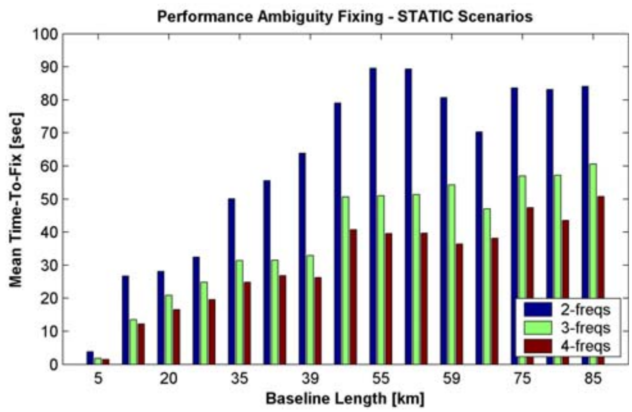

Figure 12: Galileo Fixing Performance (mean-Time-to-Fix) vs. baseline length

d) Positioning Performance vs. Baseline length

In Figure 13 and Figure 14 not much baseline dependency can be observed in correspondence with the observed positioning accuracy. As expected the positioning performance is influenced greatly by the level of carrier-phase multipath and by the number of carriers. The latter is due to a higher number of observations to determine the position.

It can be seen clearly that the number of carrier frequencies has a significant impact. Off course here the *“old engineering principle”* holds strong that the more original observations are available the higher is the accuracy of the final derived measurement.

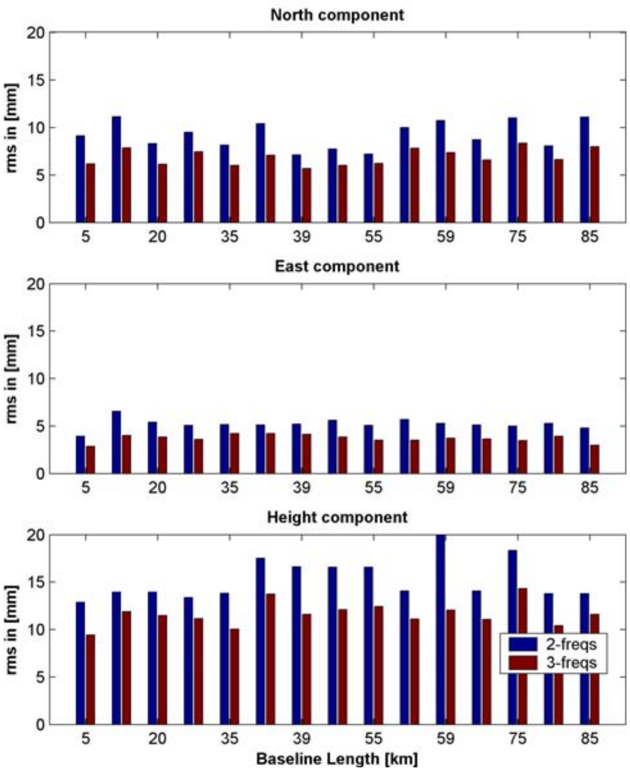

Figure 13: GPS scenarios final positioning accuracy vs. baseline length

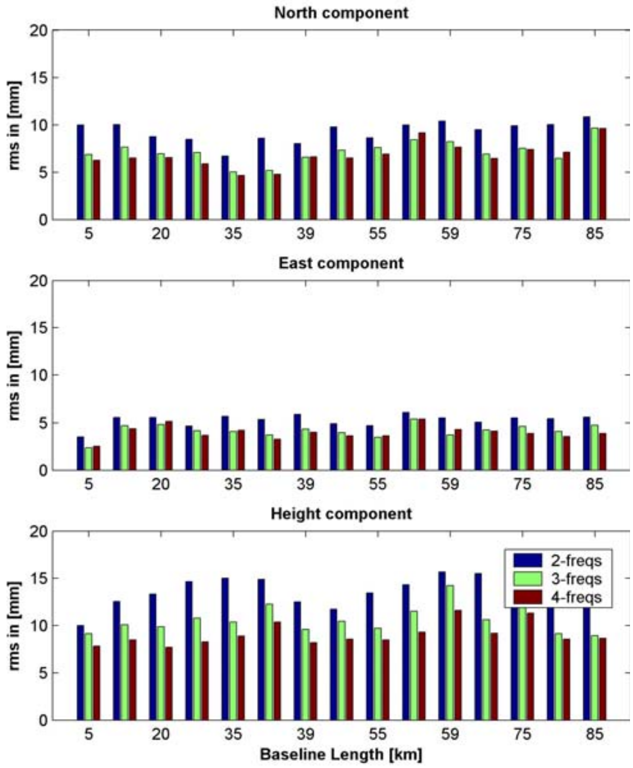

Figure 14: Galileo scenarios final positioning accuracy vs. baseline length

CONCLUSION

The obtained results of the ambiguity fixing performance analysis can be summarized as follows:

- A higher level of code noise/multipath influences greatly the fixing reliability for both GPS and Galileo scenarios.
- The correlation between Time-to-Fix and baseline length is significant for all scenarios.
- The difference in terms of fixing performance (reliability, TTF) between DCAR and TCAR is significant for all scenarios. The gain can be given in the order of magnitude of 30 percent for the reliability and the Time-to-Fix. Between the TCAR and the FCAR performance the gain is less distinct. It can be given with less than five percent for the reliability and with approximately 10 percent for the Time-to-Fix.
- To be able to strengthen the evidence for the statements above significantly longer observation periods have to be analyzed.

To close the experimental results and to give an outlook a test has been performed to predict fixing reliability/ failure rate for a combined GPS/Galileo system. The shown probability values have been computed using combined GPS/Galileo observation and orbit files. For this purpose neither a float solution nor ambiguity fixing was performed. This would have required the implementation of a second reference satellite for the second satellite system, exceeding the scope of the TCAR II experiment by far.

The figure shows a significant gain for the combination of GPS and Galileo. The predicted failure rate at least one order of magnitude lower than for the combined system.

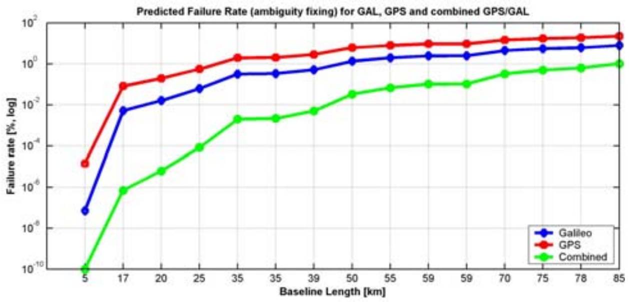

Figure 15: Predicted Failure rate for GPS, Galileo and a combined GPS/Galileo system.

ACKNOWLEGDEMENTS

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