Three or Four Carriers – How Many are Enough?

Ulrich Vollath, *Trimble Terrasat GmbH*
Knut Sauer, *Trimble Terrasat GmbH*
Francisco Amarillo, *ESTEC*
Jorge Pereira, *Socratec NavSat, S.L.*

BIOGRAPHY

Dr. Ulrich Vollath received a Ph.D. in Computer Science from the Munich University of Technology (TUM) in 1993. At Trimble Terrasat - where he is working on GPS algorithms since more than ten years - he is responsible for the Algorithm Development team. His professional interest is focused on high-precision real-time kinematic positioning and reference station network processing.

Knut Sauer holds a degree in Geodesy and Mining Surveying from the Technical University Freiberg. Recently he finished his PhD studies in Satellite Geodesy at Imperial College London. 2003 he joined Trimble Terrasat as software and development engineer where he is working on high precision kinematic positioning using the future Galileo system.

Francisco Amarillo is the ESA/ESTEC technical officer responsible for the investigation on four carrier ambiguity resolution.

Mr. Jorge Pereira Carlos is Project Scientist by Socratec NavSat, S.L. During the last two years he has been involved in international satellite navigation projects, contributing on the implementation of algorithms and software for GNSS data collection, simulation and analysis tools, as well as on the functionality and performance verification of GNSS-based systems and advanced train localization systems.

ABSTRACT

The use of three carriers to perform high precision carrier phase based positioning has been widely accepted now for the modernized GPS as well as for the planned Galileo satellite navigation system. In principle, instantaneous (one-epoch) ambiguity resolution becomes feasible for a broad range of applications. A boost of system availability and reliability is recognized as well.

For Galileo, in addition to the open access service (OS) a commercial service (PRS) is planned. Besides other improvements a fourth carrier frequency E6 will be available.
It has to be proved that the corresponding fees are justified by the actual Quality of Service.

Theoretical analyses (ADOP analyses) on the impact of a fourth frequency on the ambiguity resolution reliability and availability are presented. An increase of 1 to 15 orders of magnitude is predicted for the reliability of four-carrier single-epoch ambiguity resolution with respect to using three carriers only.

Based on a contract with the European Space agency (ESA) European Space Research & Technology Center (ESTEC), an extensive hardware simulation of the planned three- and four-carrier options has been performed. The paper provides a detailed description of this experiment. The simulation set-up consists of a modified satellite signal simulator and a prototype receiver capable of tracking more than two carriers. Ambiguity resolution and positioning were computed using the latest state-of-the-art multi-carrier algorithms.

The second subject of the experiment assesses the differences between modernized GPS and Galileo in terms of the signal structure. Galileo makes extensive use of Binary Offset Carrier (BOC) instead of Bi-Phase Shift Key (BPSK) modulation. Therefore different noise and multipath characteristics and corresponding performance indicators result.

The paper presents performance comparisons for three-carrier GPS, three-carrier Galileo and four-carrier Galileo scenarios. To cover a broad range of applications, different ionospheric and multipath conditions and varying height separations of user and reference station are covered. The user segment comprises rural vehicle, aeronautical en-route and precision approach environments.

THEORETICAL ANALYSES

One of the first possibilities to predict a satellite navigation systems performance that is not operational yet is a theoretical analysis of the ambiguity resolution success rates. Successful ambiguity resolution is a prerequisite for centimeter-level high-precision positioning.

One tool frequently used for this is the ADOP analysis. The ADOP is a measure for the average accuracy of the ambiguities computed in a floating solution to be determined for a fixed solution.

The presented analysis will be limited to the geometry-free estimation of ambiguities. The main reason is that this focuses on signal design issues, leaving out the satellite constellation und user mission parameters. A discussion of the differences between geometry-free and geometry-included assessments follows.

The investigations are limited to carrier-phase differential processing, targeting for centimeter-level accuracy positions after an initialization phase (integer ambiguity resolution). This paper focuses on instantaneous ambiguity resolution, i.e. ambiguity resolution using only one epoch of data. The success rates – the probability of getting a correct solution – will be analyzed for the case of two, three and – in the case of Galileo – four carrier frequencies available. Analyses for the hardware simulation experiment described later are added to show how much the different signal design caused by hardware limitations can be expected to bias the results.

SIGNAL DESIGNS

The following signal designs (Table 1) are used for modernized GPS ([Cliatt 2003]), the upcoming Galileo system ([Hein et. al. 2002]) and the hardware simulation performed.

System	Carrier 1 MHz	Carrier 2 MHz	Carrier 3 MHz	Carrier 4 MHz
GPS	L1 1575.42	L2 1227.6	L5 1176.45	n.a.
Galileo	L1 1575.42	E5a 1176.45	E5b 1207.14	E6 1278.75
Simulator	L1 1575.42	L2 1227.6	G(7) 1605.9375	G(24) 1615.5

Table 1: Signal designs investigated

The a priori errors were considered constant with respect of the elevation angle. Though it is known that usually multipath and ionosphere are elevation dependent, this is handled as geometry dependency for this section and postponed to geometric analyses.

For the noise numbers an average performance was assumed for all carrier phase measurements. The signal modulation and bandwidth has a limited influence of the multipath impact on the carrier phase measurements. A value of 0.01 cycles 1 sigma is very realistic compared with current GPS data from geodetic-quality receivers. The carrier phase tracking noise can be safely neglected for this analysis.

The code errors can be estimated from existing data for the GPS L1 C/A code case, too. Here, typically a noise of 15 cm and a multipath contribution of 20 cm will be present. For the other modernized signals, a performance comparing to GPS P-code has been used.

Theoretical analyses of ESTEC regarding the code multipath are used for all signals. For the noise component, a value of 5 cm can be expected for the modernized signals including GPS civil L2 (Table 2).

System	Code 1 Mpath [m] Noise [m]	Code 2 Mpath [m] Noise [m]	Code 3 Mpath [m] Noise [m]	Code 4 Mpath [m] Noise [m]
GPS	0.21 0.15	0.21 0.05	0.21 0.05	n.a.
Galileo	0.21 0.03	0.21 0.10	0.21 0.10	0.05 0.02
Simulator	0.21 0.05	0.21 0.10	0.21 0.10	0.05 0.05

Table 2: Code error specification

For the instantaneous case, the variances of code noise and code multipath are simply added as the different correlation times don't matter for one epoch of data.

ESTIMATION MODEL

The estimation model used is the following geometry-free approach:

$$\begin{pmatrix} \rho_1 \\ \vdots \\ \rho_{nf} \\ \phi_1 \\ \vdots \\ \phi_{nf} \\ I0 \end{pmatrix} = \begin{pmatrix} 1 & \frac{\lambda_1^2}{\lambda_1^2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{\lambda_{nf}^2}{\lambda_1^2} & 0 & \cdots & 0 \\ \frac{1}{\lambda_1} & -\frac{\lambda_1}{\lambda_1^2} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\lambda_{nf}} & -\frac{\lambda_{nf}}{\lambda_1^2} & 0 & \cdots & 1 \\ 0 & 1 & 0 & \cdots & 0 \end{pmatrix} \cdot \begin{pmatrix} R \\ I \\ N_1 \\ \vdots \\ N_{nf} \end{pmatrix} + \begin{pmatrix} v\rho_1 \\ \vdots \\ v\rho_{nf} \\ v\phi_1 \\ \vdots \\ v\phi_{nf} \\ 0 \end{pmatrix} = A \cdot \vec{x}$$

with the noise matrix

$$Q = E\left[\begin{pmatrix} v\rho_1 \\ \vdots \\ v\rho_{nf} \\ v\phi_1 \\ \vdots \\ v\phi_{nf} \\ 0 \end{pmatrix}^T \cdot \begin{pmatrix} v\rho_1 \\ \vdots \\ v\rho_{nf} \\ v\phi_1 \\ \vdots \\ v\phi_{nf} \\ 0 \end{pmatrix} \right] = \begin{pmatrix} \sigma\rho_1^2 & & & & & 0 \\ & \ddots & & & & \\ & & \sigma\rho_{nf}^2 & & & \\ & & & \sigma\phi_1^2 & & \\ & & & & \ddots & \\ & & & & \sigma\phi_{nf}^2 & \\ 0 & & & & & \sigma I0^2 \end{pmatrix}$$

where nf is the number of frequencies available, ρ_i and φ_i are the pseudorange and carrier measurements on carrier i and λ_i is the wavelength of carrier i.

I0 is an a priori knowledge of the ionospheric residual, normally set to 0. Its purpose is to constrain the ionospheric residual by use of the variance $\sigma I0^2$.
The ionosphere variance is used here instead of the common baseline length number as the actual ionospheric influence varies strongly with time-of-day, season, latitude and position in the solar activity cycle. Still, for a given scenario, a good knowledge of the relative ionosphere with regard to the baseline length can be obtained, usually characterized by the ionosphere ppm measure. E.g. 10 ppm ionosphere on a 50 km baseline corresponds to 50 cm of ionosphere standard deviation. So, the x-axes of all figures following, labeled as cm of ionosphere variance, can also be interpreted as baseline length in km at 10 ppm of differential ionosphere.
The state R summarized the geometric range between satellite and receiver, the receiver and satellite clock errors and the tropospheric residual, i.e. the geometric part which is thus eliminated. The approach can be classified as an ionosphere-float model, with the extremes ionosphere fixed (0 m) and ionosphere free (infinite ionosphere variance).
The estimation is done on the double difference between user receiver, reference receiver, satellite and reference satellite.
The variance/covariance matrix of the estimation can be computed as

$$C = \left(A^T \cdot Q^{-1} \cdot A \right)^{-1}$$

For the analyses in this paper, the variances have been multiplied with a factor of 2 to reflect the use of double differences instead of single raw measurements (one satellite/one receiver).

ADOP

The Ambiguity Dilution of Precision ADOP is defined as:

$$ADOP(C) = \sqrt{\det C}^{\frac{1}{n}}$$

It can be computed very efficiently by using the Eigenvalues of the variance/covariance matrix or the conditional variances of the ambiguities, see ([Teunissen et.al. 1997]).
The success rate for a given ADOP values can be computed as ([Teunissen 1998]):

$$P(ADOP) = \left(2 \cdot \Phi\left(\frac{1}{2 \cdot ADOP} \right) - 1 \right)^n$$

with $\Phi(x)$ the probability density function of the normal distribution.
The results presented are based on the assumption that at least 4 double differences have to be fixed to allow useful positioning. For that reason, the fourth power of the ADOP probabilities has been used.

$$P_{nsat} = P_{1sat}^{\ nsat}$$

The success rate can be interpreted in two ways. Primarily, it gives the probability is ambiguity resolution can be successfully performed, which is an availability measure for precise positioning. In practice, it also represents the reliability of correct ambiguity resolution for the analyzed case.
Please note that this is neither an upper nor a lower bound for the probability of successful ambiguity resolution. The value is in-betweens both bounds. As it is very convenient to compute and has proven to give realistic prediction (e.g. use for validation) it can still be used for relative comparisons of different signal scenarios.
The following figures are all shown as "Nines" characteristics. For a given success rate P, the "Nines" are computed as:

$$Nines(p) = -\log_{10}\left(1 - p\right)$$

The background of this measure is that it is very illustrative. A three nines number specifies a success probability of 99.9 percent, six nines yield 99.9999 %.

THE "NINES"

Figure 1 shows the "Nines" for the success rates of the dual-frequency scenarios.

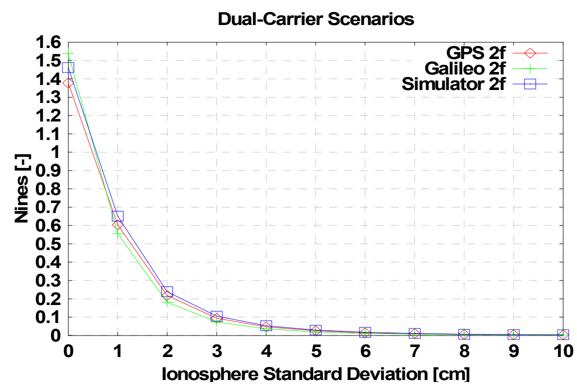

Figure 1: Dual frequency results

All systems have a quite comparable performance starting with little more than 1 "Nine", i.e. 90 % probability.
A similar situation is found for three carriers (Figure 2). The differences in signal structure and noise expectations don't result in a significantly varying performance.

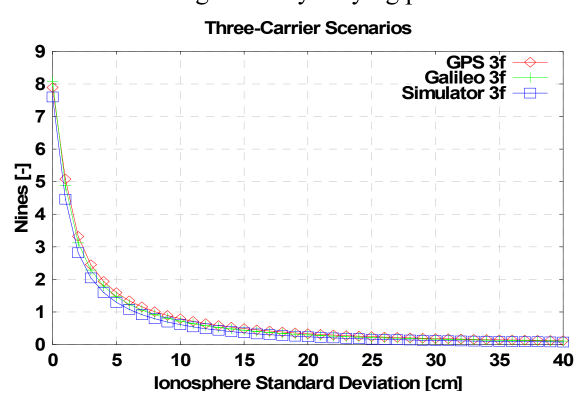

Figure 2: Three frequency results

For four carriers (Figure 3), only Galileo and the Simulator are of interest. Due to the close placement of the fourth frequency to the third in the simulator case, the predicted Galileo performance is slightly but clearly better than the simulated one.

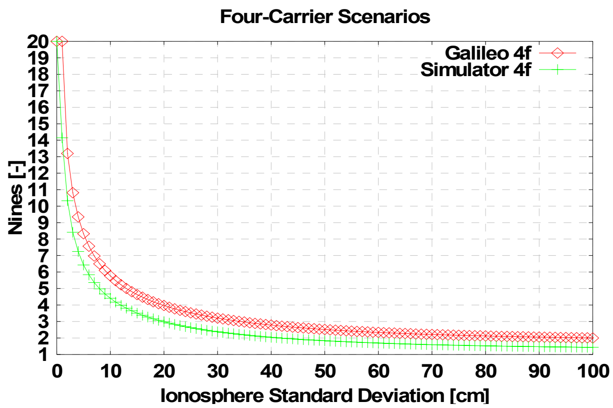

Figure 3: Four frequency results

This is very important for the final results as it shows that conservative estimates for the Galileo case will be derived with the simulation.

To visualize the difference between the success rates for different numbers of carrier, the Figure 4 shows all results together.

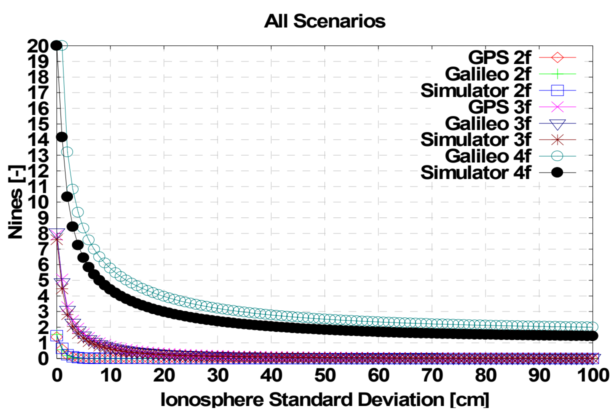

Figure 4: All frequency results

The improvements from two to three and even more from three to four frequencies are obvious. While the improvement from the third carrier gets less at higher ionosphere values, the fourth carrier will provide one “Nine” more even in case of high ionosphere values.

COMPARISON

The impact of additional frequencies is shown by the difference of the “Nines” between different numbers of carriers for the same system. Figure 5 gives the differences for dual/three frequency GPS, Galileo and Simulator.

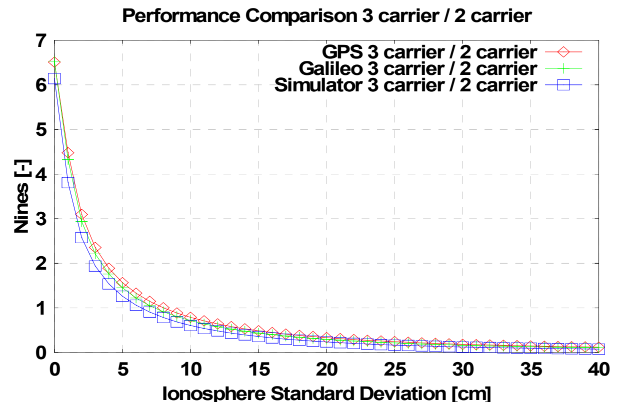

Figure 5: Comparison two carriers vs. three carriers

The improvements by use of a third carrier are very similar for all systems.
Three/four carrier comparison for Galileo and Simulator are shown in Figure 6.

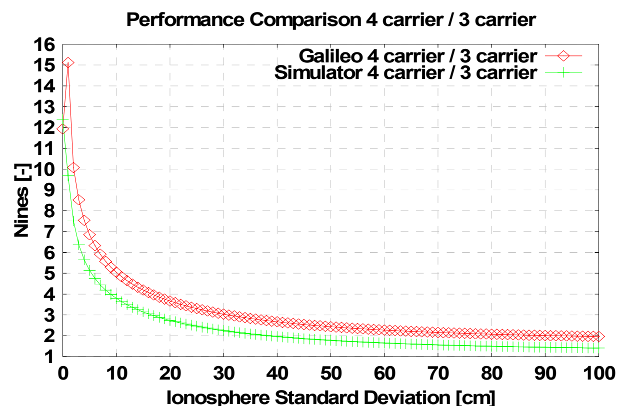

Figure 6: Comparison three carriers vs. four carriers

The simulator data profits less from the fourth carrier due to its close placement to the third one, another indication that the hardware simulation will return conservative estimates of the benefit of a fourth carrier.

SENSITIVITY ANALYSIS

To check the influence of the assumed noise levels on the success rates, the computations were repeated for modified variances of code and carrier.
The figures show the impact of twice the code noise, twice the carrier noise and both compared to the original signal baseline for the two, three and four carrier Galileo case.
For two carriers (Figure 7) degradation can be seen that is larger for the carrier case than for the code. If both code and carrier are degraded a much worse performance results. Anyhow, the biggest uncertainty about the errors is in the code.

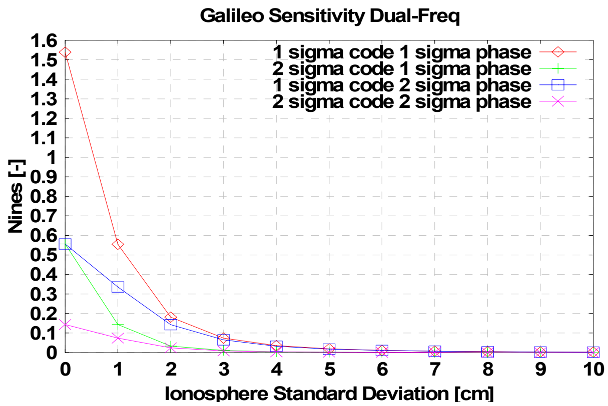

Figure 7: Error sensitivity for two carriers

For three carriers (Figure 8), the same statement holds. The success rates are much lower for worse signals with the highest influence stemming from the carrier phase errors.

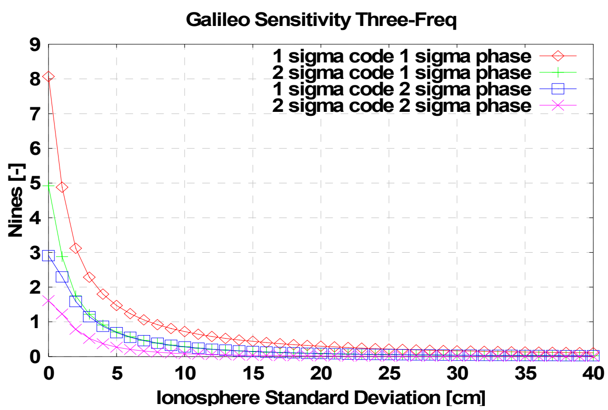

Figure 8: Error sensitivity for three carriers

For four carriers (Figure 9), too, there is a strong impact of the noise assumptions on the success rates. Still, for low ionosphere influence, high "Nines" result representing 99.9999 % reliability.

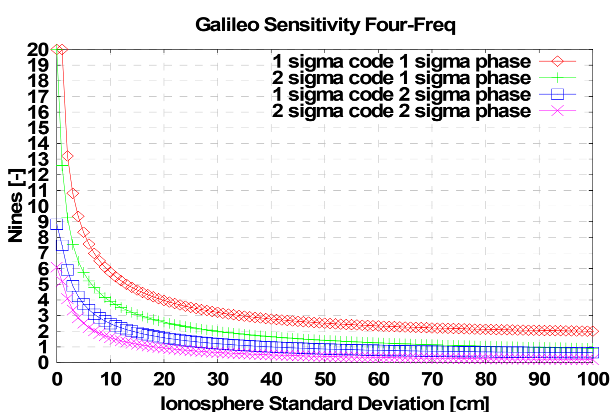

Figure 9: Error sensitivity for four carriers

One issue addressed frequently during the Galileo system design phase was the alternative of having a better second carrier design instead of adding a third carrier frequency. To investigate this using the ADOP method, Figure 10 shows the "Nines" for an improved second (E5) carrier that results in half the code and carrier variance with the current three carrier variant (L1/E5a/E5b).

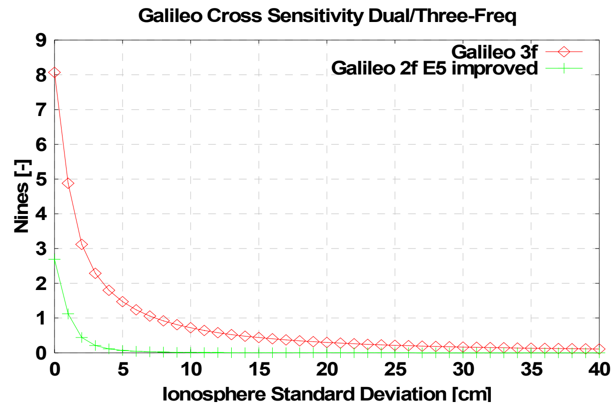

Figure 10: Comparison two (better E5) with three carriers

It can be clearly seen that the three carrier option is still much more reliable for lower ionosphere impact. Above 20 cm the two graphs converge.

INFLUENCE OF GEOMETRY

To weight the theoretical geometry-free results, the influence of the geometry has to be discussed. For the worst-case scenario of four satellites, the geometry-free results reflect the maximum performance that can be achieved as there is no overdetermination (residuals) from the geometric solution. Though this case seems quite academically, it still applies for applications with a very limited view of the sky (urban canyons, pit mines).

For "normal" applications with at least 5 satellites in view, the actual fixing performance is better than predicted from a geometry-free assessment. The additional information that all measurements refer to the same receiver position helps substantially in determining the correct ambiguities.

The influence of geometry can be quantified using the floating solution variance/covariance matrix of the minimum error carrier phase combination ([Sjoberg 1990]). A combined variance matrix can then be computed:

$$C_{comb} = \left(\vec{a}_{geo} \cdot C_{geo}^{-1} \cdot \vec{a}_{geo}^{T} + C_{g.f.}^{-1} \right)^{-1}$$

Here, C_{geo} is the variance/covariance matrix of the minimum error carrier phase geometric floating solution and $C_{g.f.}$ is the variance/covariance matrix of the complete double differenced geometry-free solution.

The corresponding ADOP predicts the success rate for the complete geometry-free and geometric combined case. If there was a simple computation method for the total ADOP from the geometry-free and geometric ADOPs, a complete assessment would be possible using the geometry-free ADOP, derived from the signal structure only, and the geometric ADOP, derived from the satellite constellation and mission parameters. Unfortunately, such a method has not been found up to this time.

One example is the short-distance dual-frequency case. Geometry-free analyses of instantaneous ambiguity resolution predict a performance that is effectively useless for practical applications. Nevertheless, with more than 7 or 8 satellites in view, a significant percentage of correct fixes (50 to 90 %) can be achieved.
With increasing number of satellites, the pure geometry-free approach will return lower success rates as all satellites have to be independently fixed, while the geometric approach usually gives a better reliability with the exception of high-noise low elevation satellites.
One fact will always hold: comparing two scenarios geometrically, the relative sequence of the success rate derived from geometry-free calculations will also hold. The absolute probabilities can vary. The verification of the theoretical comparisons of two-, three and four-carrier systems thus has to be done using a least a simulation of the geometry. The following sections describe such an experiment, simulating the receiver data for given satellite constellations.

TEST SET-UP

To retrieve performance analyses near to reality, a hardware simulation of the new signals was performed under an ESA/ESTEC contract. This project was a follow-up on previous experiments investigating three-carrier ambiguity resolution ([Vollath et. al. 1998], ([Vollath et. al. 2001]). The main purpose of the experiment was to investigate if the benefits of a fourth carrier are justifying a commercial pay-service. Also, differences in the expected performance of modernized GPS and Galileo were of interest.

For the simulations, the following equipment was used:

- A Spirent satellite signal simulator, modified to transmit GPS codes on GLONASS channels
- A breadboard receiver capable of tracking 6 satellites simultaneously on three carries.
- A atomic frequency standard
- A circuit to synchronize receiver tracking with the epoch time of the simulator
- A workstation for receiver control and data logging

The block diagram of the hardware test equipment is shown in Figure 11.
The frequencies used were described in a previous section. Especially, GPS L5 and Galileo E5b was simulated with GLONASS channel 7, Galileo E& using GLONASS channel 24. All codes were tracked as P-Codes to guarantee the low noise and multipath values required.

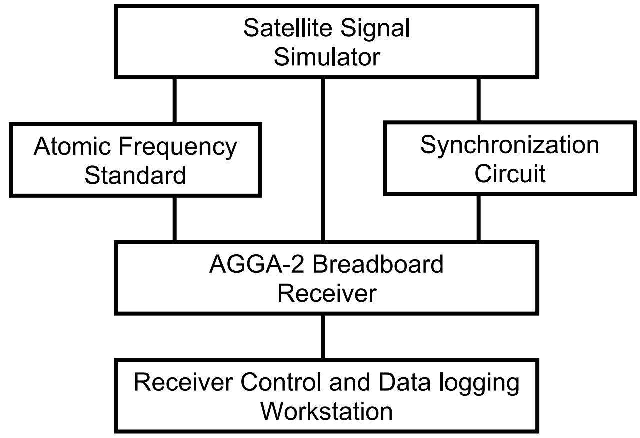

Figure 11: Test hardware set-up

To be able to generate four carriers and more that 6 satellites, every data set was combined from 4 hardware simulations. The combination of different runs requires tight synchronization of the receive times. For that reason, the atomic frequency standard was used to synchronize the signal simulator and the receiver to assure tracking at the programmed reception times. The remaining error in the receiver clocks was eliminated by the combination process. The combination procedure is shown in Figure 12.

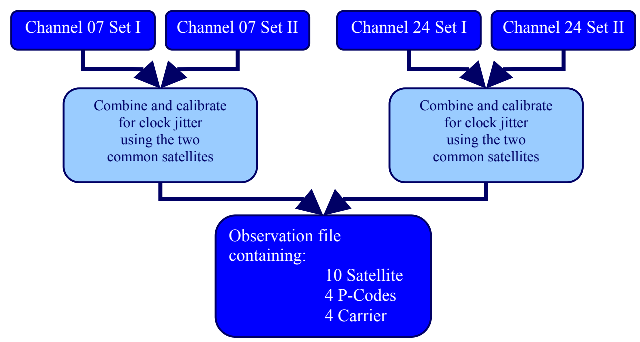

Figure 12: Combination of satellites and frequencies

TEST SCENARIOS

For the simulation, 6 static receivers were simulated with a spatial separation designed to cover an optimal range of distances between reference stations and kinematic users, up to 85 km. The kinematic receivers were simulating the following user scenarios:

- Three surface users (cars)
- Three en-route air trajectories
- One airplane precision approach

Figure 13 gives an overview of the generated receiver positions.

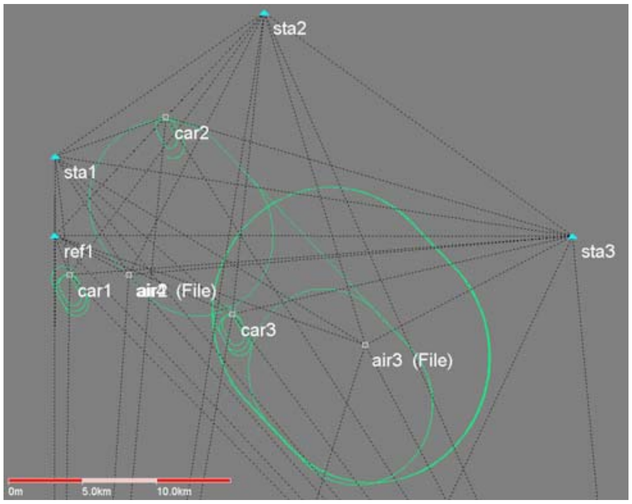

Figure 13: User scenarios

The precision approach trajectory is shown in Figure 14.

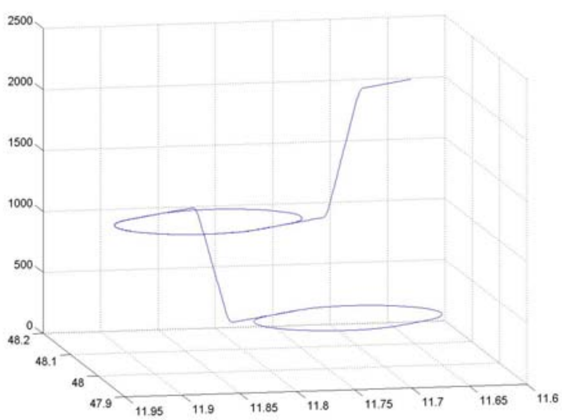

Figure 14: Precision approach trajectory

The generated power levels were adjusted so that the code errors in the simulated data contained about 21 cm (one sigma) multipath and 5 cm noise.

For the ionosphere, a broadcast GPS ionosphere message (Klobuchar parameters) was used that provided an average ionosphere influence for the generated data of about 4 ppm.

FIRST RESULTS

(to be added)

ACKNOWLEDGEMENTS

This work has been performed under ESA/ESTEC contract. Furthermore, the authors would like to acknowledge the work of Wolfgang Zsalcsik (Austrian Aerospace) during the hardware set-up and validation phase.

FUTURE WORK

The next step is to perform the complete three and four carrier ambiguity resolution analyses using the simulated data. This will shed some light on the usefulness of ADOP analyses in the geometry-free domain. The main results will be performance predictions for modernized GPS and Galileo. They will include processing with more than one epoch of data and thus provide Time-To-Fix statistics for all scenarios.

Another extension of the work done is to process the GPS and Galileo data together in one processing step. This will show the impact of having about twice as many satellites available. This would be interesting in the context of blocked satellite signals (urban canyon), too.

CONCLUSIONS

At this stage of the project the significant impact of the number of carriers on the success rates can be highlighted. Assuming good noise qualities of the signals with respect to noise and multipath, a substantial benefit from the use of a fourth carrier can be anticipated. Though more satellites generally help, an upgrade of a satellite navigation system to more carriers is probably easier (in cost terms) than to launch more satellites.

- Modernized GPS and Galileo will benefit in the same way from the third carrier. In the next couple of decades, the maximum performance will probably be achieved with a combined GPS three-frequency plus Galileo four-frequency system.
- Whether the biggest impact will come from the combination of both systems or from the fourth carrier remains to be analyzed in detail.
- The theoretical analyses have shown that the hardware simulation results will be applicable to the real data. In the four carrier case they provide a conservative estimate of its benefits. A validation using simulated data will show if the predicted differences will be reflected by the use of real (or almost real simulated) data in the geometric domain.
- Three carriers will be enough when comparing today's dual-frequency systems with the new three-carrier positioning. Four carriers are enough when the reliability should be very high, especially in cases of substantial ionospheric influence.

In addition, the old truth still holds in the upcoming and modernized systems: reducing the multipath by good antenna and receiver designs and careful signal design will always help.

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